THE COLLEGE OF AERONAUTICS
CRANFIELD

THESIS

TITLE A PROPOSAL FOR HELICOPTER VIBRATION REDUCTION USING ELECTRO-HYDRAULIC SERVO CONTROL

NAME D. J. TAFT

DATE OCT. 1967

A PROPOSAL FOR

HELICOPTER VIBRATION REDUCTION

USING ELECTRO-HYDRAULIC SERVO CONTROL

by

Denis J. Taft

Lieutenant Commander

United States Navy

Department of Electrical and Control Engineering

The College of Aeronautics

Cranfield, England

October, 1967

Thesis
1967
TAFT, D.

ABSTRACT

This thesis is an attempt to prove feasible a new electro-hydraulic servo control system designed to reduce vertical vibrations of a helicopter. A simple theoretical investigation points out the source of the rotor-induced vibrations. A brief resumé then reviews past and present attempts to alleviate helicopter vibrations and mentions pending future proposals.

The intended proposal is described for the general helicopter with a conventional rotor. The closed loop transfer function for the proposed servo system is then obtained for one particular case of a typical helicopter. The system as proposed appears feasible, however, further theoretical work is necessary, followed by some type of simulation.

This thesis was done as an individual project in conjunction with a course in Flight Control at the College of Aeronautics, Cranfield, England.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

* Note, Fig. 10 is out of proper numerical order.

LIST OF SYMBOLS/ABBREVIATIONS

Symbol	Definition
a	lift curve slope for a blade
$(a_1)_0$	steady state long. flapping inclination from tip-path plane
a_λ, b_λ	blade flapping motion coefficients
α	angle between direction of flight and rotor disc plane
α_r	absolute angle of attack of a blade element
$(\alpha_1)_0$	steady state long. rotor inclination
B	tip loss factor, R_e/R
B_1	Lateral cyclic pitch applied
b	number of blades
$b_{n_\lambda}, c_{n_\lambda}$	thrust coefficients
β	blade flapping angle
β_0	coning angle (upward tilt of all blades)
$\dot{\beta}$	blade flapping velocity
$\ddot{\beta}$	blade flapping acceleration
c	chord of a blade element
c_l	blade section lift coefficient
c_d	blade section profile drag coefficient
c.g.	center of gravity
C_T	thrust coefficient, $T/\rho\Omega^2\pi R^4$

Symbol	Definition
C_{mf}, C_t	non-dimensional moment coefficients for fuselage and thrust
dL	local lift at a blade element
dD	local drag at a blade element
dr	elemental blade length
F	total force acting on a blade
F_y	component of total force F, acting in Y direction
F_z	component of total force F, acting in Z direction
g	acceleration (used in text as vibration level unit)
Hz	Hertz (cycles per second)
I_P	Mom. of inertia about the flapping hinge
K_a	accelerometer gain term
K_β	constant relating β_o and acceleration, i.e. Accln = $K_\beta \times \beta_o$
K_F	Filter gain term
K_{FB}	normalised feedback gain
K_H	hydraulic actuator gain term
K_θ	constant relating θ_o and acceleration, i.e. $\theta_o = K_\theta \times$ Accln
m	blade mass
M_T	moment about flapping hinge due to airloads
M_W	moment about flapping hinge due to blade weight

Symbol	Definition
r	radial distance to a blade element
R	rotor blade radius
R_e	effective rotor blade radius
RPM	revolutions per minute
s	Laplace transform variable for d /dt
S_e	πR_e^2 , effective rotor disc area
T	rotor thrust
T_λ	time constant or period (seconds)
T_N	inherent rotor thrust pulsations
U_R	total velocity component in the X direction at a blade element
U_T	total velocity component in the Y direction at a blade element
U_P	total velocity component in the Z direction at a blade element
v	axial component of induced velocity at a blade element
V'	resultant air velocity at the rotor
x	r /R
$\dot{x}, \dot{y}, \dot{z},$	component velocities at a blade element
θ	blade section pitch angle from zero lift chord line
θ_0	undisturbed blade collective pitch
θ_D	blade collective pitch demanded
ϕ	induced angle of attack at a blade element

Symbol	Definition
ϕ_N	rotor thrust pulsations due to turbulence, gusts, etc.
ρ	air density
λ	$(V\sin\alpha - v)/\Omega R$, mean inflow factor
λ_a	$-v/\Omega R$
Ω	rotor angular velocity
μ	tip speed ratio, $V\cos\alpha/\Omega R$
$(\mu_z)_0$	steady state vertical velocity $\dot{z}/\Omega R$
ψ	angular displacement about the Z axis of the ZX plane from its initial position, azimuth angle of a blade
ω_{n_i}	resonant frequency of component i
ζ_i	damping ratio of component i

1. INTRODUCTION

Helicopter vibrations have always been an important issue to helicopter designers and engineers. Recently, as more and more helicopters are being used commercially and militarily, the problem of helicopter vibrations has increased in importance. Recent studies have shown that the resonant frequencies of the human body are found in the 4 - 6 Hz region in seated subjects [10]. The problem of damping very low frequencies (2 - 6 Hz) is most difficult to solve. However, vibrations at these frequencies cause the most difficulties: airsickness, fatigues, etc. High frequency vibrations appear to have more effect on the helicopter itself, and on equipment inside it.

The principal vibrations originate at the rotor head at frequencies proportional to the number of rotor blades times the rotor speed. These are due to the pulsating aerodynamic forces acting on each rotor blade. Other sources of vibration are due to the elastic properties of the blades, engine vibrations, torsional vibrations of the rotor mast, and, of course, from the tail rotor if one is installed.

Up to now people have come to tolerate these vibrations or to reduce them by detuning or isolating the rotor vibratory motion. To quote D. E. Brandt,

> With the advent of larger and faster aircraft, the magnitudes of the vibratory exciting forces are increasing while classical control through detuning and isolation is becoming increasingly difficult and penalizing weightwise. [2]

The present opinion of many people is to fold-away the rotor and concentrate on V/STOL aircraft or compound helicopters. At any rate, new ideas are necessary and are being generated.

It is the purpose of this paper to propose a new idea in the form of a servo-control to reduce helicopter vibrations. It is <u>not</u> the purpose of this paper to undeniably prove that the system will work for any or all helicopters but rather to indicate its feasibility.

The outline of the proposal is very simple. The following chapter is devoted to a conventional approach in deriving the thrust equation for a helicopter. It is included to establish a theoretical basis and also to emphasize the complexity of the rotor behaviour, and is followed by a chapter that discusses past, present, and future trends concerning helicopter vibrations.

The new vibration control system is then proposed and is discussed in some detail, making suggestions for its further development and drawing such limited conclusions that are possible from a study which has not had the advantage of an actual flight experiment.

2. HELICOPTER THEORY

One of the reasons justifying the succeeding analysis is to illustrate the complexity in even a simple approach to helicopter analysis and to show many of the assumptions necessary for simple analysis. Any more detailed study would probably require the use of high speed computers and large engineering teams. It will be shown that the vertical thrust of a helicopter is composed of pulsating aerodynamic forces.

The following quote from J. Shapiro's textbook is worthy of inclusion here.

> What is new in helicopters is the association of elements whose properties are decisively influenced by rotation with non-rotating elements. When we have succeeded in expressing the properties of the rotating elements from the point of view of the non-rotating observer the problem of determining the vibrating properties of the helicopter as a whole becomes a conventional problem [11].

This does not make it an easy problem, but it is really no more difficult than the majority of other real-life processes.

The following analysis is as found in Nikolsky's textbook [9]. The helicopter is considered to be in forward flight and has a rotor containing flapping,

feathering, and drag hinges. The first assumption is that the induced velocity v is constant over the rotor disc.

From momentum theory it can be shown that

$$v = \frac{T}{2 S_e V' \rho} \tag{1}$$

where V' is the resultant velocity acting on the rotor as shown in Fig. 1. From Fig. 1 we can write

$$V' = \left[(V \sin\alpha - v)^2 + V^2 \cos^2\alpha \right]^{1/2}. \tag{2}$$

Now the mean inflow factor is defined as

$$\lambda = \frac{V \sin\alpha - v}{\Omega R} \tag{3}$$

and the tip speed ratio is defined as

$$\mu = \frac{V \cos\alpha}{\Omega R}. \tag{4}$$

By substituting (3) and (4) into (2), V' can be expressed as

$$V' = \Omega R (\lambda^2 + \mu^2)^{1/2}. \tag{5}$$

The definition of thrust T is given as

$$T = C_T \rho \Omega^2 \pi R^4. \tag{6}$$

Se is the effective disc area and equals

$$S_e = \pi R_e^2 = \pi (BR)^2 \tag{7}$$

where B is the tip loss factor. By substituting (5), (6), and (7) into (1) we obtain

$$v = \frac{\frac{1}{2} C_T \Omega R}{B^2 (\lambda^2 + \mu^2)^{1/2}}. \tag{8}$$

Only the lift-producing aerodynamic forces on a rotor blade are to be investigated. There are, however, dynamic and gravitational forces present too. By means of illustration consider a 2000 pound helicopter supported by a two-bladed rotor. The average lift developed by a single blade is 1000 pounds and if the blade weight is of the order of 50 - 100 pounds the gravitational force can be neglected. The main dynamic force is the centrifugal force of a blade and the larger component of this is in the plane of rotation. Since the aerodynamic forces are dependent on the square of the velocity, the analysis must continue by finding the velocity at a rotor blade element. If

this velocity is periodic, then the velocity squared will generate harmonics.

Fig. 2 shows the components of velocity at a blade element at radius r due to the flapping motion of a blade with flapping velocity $\dot{\beta}$. The X' Y' Z' cartesian coordinate system is centered at the rotor hub with Z' in the direction of the rotor shaft and the X' Y' plane is the plane of the rotor disc, perpendicular to Z'. The X Y Z coordinate system has the X axis coincident with the feathering axis of the blade, the Y axis is coincident with the zero-lift line of the blade root section and perpendicular to the X axis, and the Z axis is normal to both the X and Y axes. The X Y Z system is a set of rotating axes. Hence, from Fig. 2 the components of velocity due to blade motion are

$$\dot{x} = 0 \tag{9}$$

$$\dot{y} = -r \cos\beta\, \Omega \cong -r\Omega \tag{10}$$

$$\dot{z} = -r\dot{\beta} \tag{11}$$

From Fig. 1 we have

$$\dot{X}' = V_{x'} = V\cos\alpha = \mu\Omega R \tag{12}$$

$$\dot{Z}' = V\sin\alpha - v = \lambda\Omega R \tag{13}$$

By resolving the $\dot{X}'$ component into components perpendicular and parallel to the plane of the rotor disc, as shown in Fig. 3, the velocity $\dot{y}$ due to $\dot{X}'$ is found as

$$\dot{y}_{\dot{x}'} = -V\cos\alpha\sin\psi = -\mu\Omega R\sin\psi \tag{14}$$

The other component is in the ZX plane and is equal to V cos α cos ψ. ψ is the azimuth angle of a blade.

Fig.4 shows the two components, V cos α cos ψ and V sin α - v, in the Z X plane and the respective X and Z velocity components. From Fig. 4

$$\dot{X}_{\dot{x}'} = V\cos\alpha\cos\psi\cos\beta = \mu\Omega R\cos\psi\cos\beta \tag{15}$$

$$\dot{X}_{\dot{z}'} = (V\sin\alpha - v)\sin\beta = \lambda\Omega R\sin\beta \tag{16}$$

$$\dot{Z}_{\dot{x}'} = -V\cos\alpha\cos\psi\sin\beta = -\mu\Omega R\cos\psi\sin\beta \tag{17}$$

$$\dot{Z}_{\dot{z}'} = (V\sin\alpha - v)\cos\beta = \lambda\Omega R\cos\beta \tag{18}$$

As β is quite small another standard approximation is to let $\sin\beta = \beta$ and $\cos\beta = 1.0$. Thus, adding all components together gives

$$\dot{x} \equiv U_R = \mu\Omega R\cos\psi + \lambda\Omega R \tag{19}$$

$$\dot{y} \equiv -U_T = -r\Omega - \mu\Omega R\sin\psi \tag{20}$$

$$\dot{z} \equiv U_P = -r\dot{\beta} - \mu\Omega R\beta\cos\psi + \lambda\Omega R \tag{21}$$

Still following Nikolsky's (Ref. 8) analysis, the angle of attack at a blade element equals

$$\alpha_r = \Theta + \phi . \tag{22}$$

Θ is a function of root incidence, blade twist, blade azimuth (hence periodic), cyclic and collective pitch, and hinge geometry. ϕ is the $\tan^{-1} U_P/U_T \cong U_P/U_T$.

The local lift and drag forces along a blade can be shown to equal

$$dL = \tfrac{1}{2} \rho c c_\ell U^2 dr \tag{23}$$

$$dD = \tfrac{1}{2} \rho c c_d U^2 dr \tag{24}$$

If these are now resolved into Y and Z components, as shown in Fig. 5, the result is

$$\frac{dF_y}{dr} = \frac{dL}{dr} \sin\phi - \frac{dD}{dr} \cos\phi \tag{25}$$

$$\frac{dF_z}{dr} = \frac{dL}{dr} \cos\phi + \frac{dD}{dr} \sin\phi \tag{26}$$

Equations (25) and (26) show one of the cross-coupling of forces normal to and in the plane of the rotor disc.

Continuing with the vertical force equation only, another standard assumption to make is that $\cos\phi \cong 1$, $\sin\phi \cong \phi$, and $\left(\frac{dD}{dr}\right)\phi \cong 0$. This leaves

$$\frac{dF_z}{dr} = \frac{dL}{dr} . \tag{27}$$

Substituting equation (23) into (27) gives

$$\frac{dF_z}{dr} = \tfrac{1}{2} \rho c c_\ell U^2 \cong \tfrac{1}{2} \rho c c_\ell U_T^2 . \tag{28}$$

Now $C_\ell = a\alpha_r$ where a is the slope of the lift curve for the blade and $\alpha_r = \Theta + \phi \cong (\Theta + {}^{U_P}/_{U_T})$. We can write (28) as

$$\frac{dF_z}{dr} = \tfrac{1}{2}\rho c a\left(\Theta + {}^{U_P}/_{U_T}\right)U_T^2. \qquad (29)$$

At this point the term β is to be defined. The differential equation of motion of the blade may be obtained by taking the sum of the moments about the flapping hinge.

$$\Sigma M = 0 = \Omega^2 \sin\beta\cos\beta\int_0^R mr^2 dr + \ddot{\beta}\int_0^R mr^2 dr - M_T + M_W \qquad (30)$$

Mw is the moment due to the weight of the blade and is usually so small that it is neglected. M_T is the moment due to airloads. The basic airloads are now known to be harmonic by virtue of expressions (20), (21), and (29), as the velocity squared terms will generate harmonics, hence M_T is a harmonic function of ψ. With the usual small angle approximations and $\int_0^R mr^2 dr = I$, the moment of inertia of the blade,

$$(\ddot{\beta} + \beta\Omega^2)I - M_T(\psi) = 0. \qquad (31)$$

The steady state solution of this equation is a harmonic function in ψ and can be expressed in terms of the Fourier series

$$\beta = \beta_0 - a_1 \cos\psi - b_1 \sin\psi - \cdots - a_n \cos n\psi - b_n \sin n\psi . \tag{32}$$

β_0 is the built in coning angle and the a_i and b_i are Fourier coefficients. For this analysis higher order terms will be neglected, giving

$$\beta = \beta_0 - a_1 \cos\psi - b_1 \sin\psi \tag{33}$$

$$\dot{\beta} = \left[a_1 \sin\psi \frac{d\psi}{dt} - b_1 \cos\psi \frac{d\psi}{dt}\right] = \Omega (a_1 \sin\psi - b_1 \cos\psi) \tag{34}$$

Substitute these expressions into (20) and (21); at the same time let $x = r/R$, leaving

$$U_T = \Omega R \left[x + \mu \sin\psi\right] \tag{35}$$

$$U_P = \Omega R \left[\lambda - x(a_1 \sin\psi - b_1 \cos\psi) - \mu \cos\psi(\beta_0 - a_1 \cos\psi - b_1 \sin\psi)\right] \tag{36}$$

Equation (36) can be simplified to,

$$U_p = \Omega R\left[\lambda + \tfrac{1}{2}\mu a_1 + (-\mu\beta_0 + x b_1)\cos\psi - x a_1 \sin\psi\right.$$

$$\left. + \tfrac{1}{2}\mu a_1 \cos 2\psi + \tfrac{1}{2}\mu b_1 \sin 2\psi\right] \qquad (37)$$

If the expressions (35) and (37) are substituted into (29), keeping $x = r/R$, the result is the expression for the incremental thrust at a blade element. This is quite clearly a harmonic function.

$$\frac{dF_z}{dx} = \pm \tfrac{1}{2}\rho a \Omega^2 R^3 c\left[\left(\frac{\theta\mu^2}{2} + \lambda x + \theta x^2\right)\right.$$

$$+ \left(b_1\frac{\mu^2}{4} - \beta_0\mu x + b_1 x^2\right)\cos\psi$$

$$+ \left(\lambda\mu + \frac{a_1\mu^2}{4} + 2\theta\mu x - a_1 x^2\right)\sin\psi$$

$$+ \left(-\theta\frac{\mu^2}{2} + a_1\mu x\right)\cos 2\psi + \left(-\beta_0\frac{\mu^2}{2} + b_1\mu x\right)\sin 2\psi$$

$$\left. - \tfrac{1}{4}\mu^2 b_1 \cos 3\psi + \tfrac{1}{4}\mu^2 a_1 \sin 3\psi\right]$$

$$(38)$$

The minus sign indicates a region of flow reversal.

The average thrust is

$$T = bF_z \qquad (39)$$

where b equals the number of blades and F_z is obtained by integrating (38) from blade tip to root and from 0 to 2π.

$$F_z = \frac{1}{2\pi}\int_0^{2\pi} d\psi \int_0^{B} \left(\frac{dF_z}{dx}\right) dx - \frac{2}{\pi}\int_{\pi}^{2\pi} d\psi \int_0^{-\mu\sin\psi} \left(\frac{dF_z}{dx}\right) dx \tag{40}$$

where B is, as before, equal to R_e/R. The second term is due to the effect of flow reversal. This flow exists on the side of the rotor disc from $\psi = \pi$ to $\psi = 2\pi$. The extent of flow reversal along a blade is found by setting (35) equal to zero, giving the region from $X = 0$ to $X = -\mu\sin\psi$. This region is shown in Fig. 6.

The above integration will not be performed here.. At this point it should be evident that from expression (38) the thrust may instead be expressed by

$$T = a + \sum b_n \sin n\psi + \sum C_n \cos n\psi \, . \tag{41}$$

Thus, for a rotor with b blades

$$T = a + \sum b_{n_1} \sin n\psi + \sum b_{n_2} \sin n\left(\psi + \frac{2\pi}{b}\right) + \cdots + \sum b_{n_b} \sin n\left[\psi + (b-1)\frac{2\pi}{b}\right] + \sum C_{n_1} \cos n\psi + \sum C_{n_2} \cos n\left(\psi + \frac{2\pi}{b}\right) + \cdots + \sum C_{n_b} \cos n\left[\psi + (b-1)\frac{2\pi}{b}\right] \tag{42}$$

Thus it can be seen that inherent in the thrust of a helicopter there exists a definite pulsatance. This pulsatance is the primary cause, or forcing function, of the vibrations we wish to reduce. The coefficients have the dimensions of thrust and can be obtained by integrating (38) from blade tip to root.

A slightly more physical understanding of rotor vibration is as described by Prof. J. A. J. Bennett [1]. The centroid of the area of lift or thrust distribution curve does not in general coincide with the point of application (C) of the inertia forces (Fig. 10). The blade therefore carries a

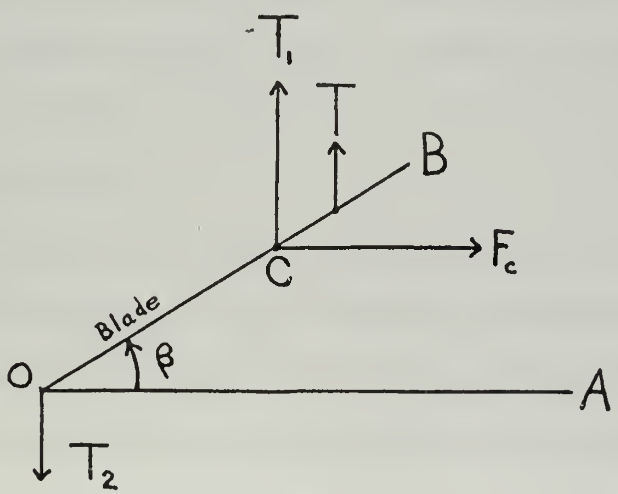


Fig. 10 Simple Blade Force Location Diagram

bending moment which produces a shear force (T_2) at the

flapping pivot (O). The blade thrust T may be replaced by the components T_1 and T_2, the former acting at the point of application of the resultant (Fc) of the inertia forces. T_1 and Fc produce a tensile force acting along the blade, the flapping angle of which is given by $\tan \beta = \frac{T_1}{F_c}$.

Now, as the blade rotates, T varies in both amplitude and location, thus T_1 and T_2 are variable, giving rise to axial or vertical vibration, the two components of which may be designated "tensile vibration" and "shear vibration". As the tensile vibration (T_1) is due to variation of the flapping angle β, this component may also be referred to as "flapping vibration" and as the shear vibration (T_2) is associated with bending in the blade, it may also be referred to as "bending vibration".

The contributions of a single blade to flapping and bending vibrations can be analysed harmonically into Fourier series whose terms have periodicities of once, twice, and so on per revolution of the rotor and the corresponding terms of the series for the several blades will be separated by phase angles, proportional to the angular spacing of the blades. On adding together

the series for the several blades to obtain the total axial components of vibration, all the terms, except those whose frequency is an integral multiple of the fundamental frequency, i.e., the rotational speed of the rotor, multiplied by the number of blades, disappear.

As Ref. 1 points out, flapping vibration is therefore due to the simultaneous upward displacement of the blades together, or in other words, due to the periodic coning of the blades or periodic upward displacement of the tip-path plane and not caused by periodic tilting of the tip-path plane. Flapping is used to denote any periodic angular displacement of the tip-path plane whereas periodic coning is any periodic axial displacement of this plane. Bending vibration can be minimised by making the blades longitudinally flexible so that the bending due to the periodic longitudinal excursion of T produces internal deflection in the blade, thereby preventing the transmission to the blade root of the periodic shear force T_2.

The preceding analysis and discussion is by no means unique. J. P. Jones, for example, derives the vertical force as a shear S_N at the rotor hub, due to blade bending [5]. His result, for a 2-bladed

Cheeseman rotor, is

$$S_N = 2\frac{(EI)_o}{R^2}f'''(0)\,\gamma^2(a_{22}\cos 2\psi + b_{22}\sin 2\psi) \tag{43}$$

where this notation is explained in Ref. 5, and ψ has the same meaning as used in this paper. Another approach to deriving the equations of harmonic air loading has been done by R. H. Miller of MIT. He used circulation theory combined with downwash and wake effects to give a similar answer [8]. The preceding analysis was presented in some detail. Several of the expressions and simplifying arguments will be referred to in later discussion.

Future analyses will probably require a computer approach right from the start. J. P. Jones has already presented excellent reasons for the use of an analog computer to calculate rotor blade motion [6]. In Ref. 6 he discusses the need for an analog-digital integrated (hybrid) computer and gives a sample solution to the blade flapping equations.

3. SOME HELICOPTER VIBRATION CONTROL METHODS

The subject of past, present, and future forms of helicopter vibration control are discussed in this paper to present the new proposal in a suitable context. Helicopters, auto-giros, gyrodynes, and many other forms of rotorcraft have been with us for years. But in the past vibrations were a small problem, partly because of the short time duration of flights and partly because little analysis was done to investigate the cause of helicopter vibrations. The 'build-them-stronger' ideas seemed to have prevailed so far over the 'think-them-out' ideas.

As the number of helicopters increase so do their endurance, speeds, and complexities. The result, today, is that vibration control in helicopters may be fast becoming a serious problem, and certainly a complex one. The importance of the problem was, however, realized some years ago. Passive isolation systems were among the first to appear. These attempted to isolate rotor induced vibrations mechanically from the helicopter fuselage via passive spring-mass-damper systems. There are two drawbacks to this approach.

One is that the deflections in the system are usually intolerable because of the large vertical deflections of the springs in the isolation system.. The second really significant drawback is that this system does nothing to alleviate the cause of the vibrations. The rotor still suffers from the harmonic air loading. These passive isolation systems might be more effective on purely in-plane vibrations.

Within the past few years active isolation systems have been introduced. The servo-controlled rotor vibration isolation system introduced by Smollen, Marshall, and Gabel is one [13]. Their active system consists of a pneumatic actuator which positions the base of their isolated rotor system. The actuator receives signals generated from the relative motion between the fuselage and the isolated rotor. Their result was the reduction of vibration levels in their model by factors of six and ten to one.. Their analysis included the elastic properties of the fuselage and rotor and their results alone suggest no simplifications should be made when analysing new systems. Ref. 13 also contains a statement that vibration attenuation by reduction of periodic aerodynamic forces on the rotor

blades is conceptually appealing but ineffective due to lack of knowledge about these forces.

An attempt has been made to reduce vertical fuselage vibrations, reduce oscillatory rotor loads, and delay retreating blade stall by using second harmonic control (SHC) [15]. The SHC mechanism of Ref. 15 was designed to add an oscillatory control motion occurring at twice per main rotor revolution which could be controlled in amplitude and phasing with respect to blade azimuth. A modified UH - 1A was used and both the SHC amplitude and phase could be adjusted in flight. Even though the mechanism accomplished the anticipated changes in air load 2/rev thrust pulsations, beneficial effects on vibration and load reduction were small.

A few of the conclusions of Ref. 15 are pertinent to this paper. The first conclusion is that for optimum results, the phasing and amplitude of the SHC input have to be controlled continuously with forward speed (assymetric velocity). This might prove difficult to achieve with a purely mechanical device. A second conclusion is that a helicopter with its cabin located under the mast might realize benefits from a simultaneous

reduction in vertical rotor vibrations and oscillatory rotor loads. This came about as a result of testing at 80 knots, where it was found a simultaneous reduction in vertical vibration level at the cg and reduction in oscillatory blade bending moments was coupled with increased vibrations of the forward located cabin of the test helicopter. These results apply only to the UH - 1A however. A final quote from Ref. 15 follows.

> It is shown that the state of the art of present theoretical methods is still insufficient to predict accurately complex phenomena such as are associated with the second harmonic feathering. This illustrates the need for experimental investigations of new devices and theories.

Another device tested not too long ago was a helicopter vibration indicator [7]. This system was to alert the pilot by means of a warning system to a dangerous condition of vibration, supplying him with the location, amplitude, and frequency of the critical vibrations. The pilot is then to establish the cause of the vibration and to take necessary corrective action. This type of system might be alright for an in-flight recorder, but it is doubtful that pilots could be trained to respond swiftly and

accurately to such a warning system.

As stated previously the predominant vibration frequencies are equal to whole digit multiples of the number of blades times the rotor angular velocity. Ref. 7 points out that a predominant 1/rev. vibration can also exist because of the misalignment of one blade. And it is vibration at these low frequencies which is most harmful to pilots, recalling Ref. 10. The vibration pickup used in Ref. 7 was a Schaevitz model VG - 10 accelerometer with linear response up to 40 Hz and a natural frequency of 65 Hz.

Other devices under study for future use include second and third harmonic control, pendulum absorbers for loads (PALS), a mechanical kinematic absorber (UREKA), and force balancers. These are described briefly in Ref. 2 and are more fully described in separate articles. Even so, these new ideas still attempt to control airframe vibratory loads only and not the rotor vibratory loads. Drees and Lynn discuss general vibration problems in their paper on the promise of compound helicopters [4]. They certainly point out the importance of a better understanding of the rotor in order to reduce or control helicopter

vibrations. With regard to rotor isolation they state that

> in addition to the classical spring-damping isolation systems now in use, new systems involving kinematic linkages, hydraulic or pneumatic operation can be expected to appear.

And with that remark it is appropriate to begin the new proposal.

4. A NEW SERVO-SYSTEM PROPOSAL

The system to be proposed is an instrumented electro-hydraulic servo system designed to reduce vertical vibrations of the complete helicopter by varying the collective pitch of the rotor blades as some function of vertical vibration. Since this thesis is only an initial feasibility study it does not contain results for an actual system. S. S. Sherby's paper on design philosophy of the OH - 5A helicopter takes a similar form, and likewise is intended to illustrate a new approach to an existing problem, with the object of giving others a possible start for further work on this proposal [12].

It is proposed that a vibration sensing device should be mounted near the c.g. of a helicopter with its input sensing axis vertical. This may prove difficult to do in practice, so the device may have to be installed on the rotor mast or on a part of the basic frame near the pilot's seat. The reason the sensing device is to be mounted as near to the c.g. as possible is so that it can sense as accurately as possible the true vertical acceleration being experienced by the helicopter. Electrical signals from this instrument will then pass through an amplifier and a filter and will be the input

to an electro-hydraulic servo valve. This servo valve will then control the motion of an hydraulic actuator. The actuator is to be in series with the collective pitch control and will modify the collective pitch so as to produce a lift force on the helicopter that reduces the sensed vibrations. It is intended that the actuator will not be able to apply full collective pitch. It will be able to apply only a limited amount of collective pitch compared with that which can be applied by the pilot, giving the pilot over-riding control.

The sensing device will be an accelerometer. Fortunately an accelerometer has rugged characteristics, long life, high reliability, small size, light weight, and is easy to instal. The device must detect all vibrations occuring over a rather wide frequency range. There are many commercially available accelerometers and the one chosen for this proposed system should have a linear response up to 100 Hz, requiring a natural frequency of about 150 Hz. The reason for the higher frequency response is this bandwidth will include 40 - 50 Hz at which damaging vibrations have been found to occur occasionally in some helicopters.

A filtering device of some kind has to be inserted in the feedback path of the closed loop system in order to block the manoeuvre g which the accelerometer also detects, otherwise the feedback mechanism would tend to prevent ordinary manoeuvres. However, this filter must pass all other vibration signals for the intended vibration control purpose. The manoeuvre spectrum of a helicopter is not clearly defined, but a typical high value of control response frequency is about two rad/sec. Therefore the manoeuvre g variation could be considered to contain frequencies from, say, two rad/sec to zero. Thus a high-pass filter seems to be required.

The use of hydraulic actuators on helicopters is not novel. For example, the Kaman Aircraft Corporation used a small hydraulic actuator to give control boost, automatic stabilization, cyclic stick damping, collective stick rate limitation, and gust alleviation [3]. It contained an electro-hydraulic servo valve of the type necessary for the proposed system in order to get the electrical signals to the actuator.

Enough has been written to permit drawing a first, simplified block diagram. Referring to Fig. 7, the arrow heads indicate the signal flow directions. In

the following, accelerations being referred to on the block diagrams, or in discussion, will be those along the Z axis, normal to the XY plane as defined in Chapter 2. As can be seen in Fig. 7, the harmonic vibrations enter the system as an input called T_N, which represents the inherent thrust pulsations developed on a rotor blade, as described in Chapter 2. The force inputs to the system due to wake effects, fuselage and blade interference, and turbulence will be called ϕ_N.

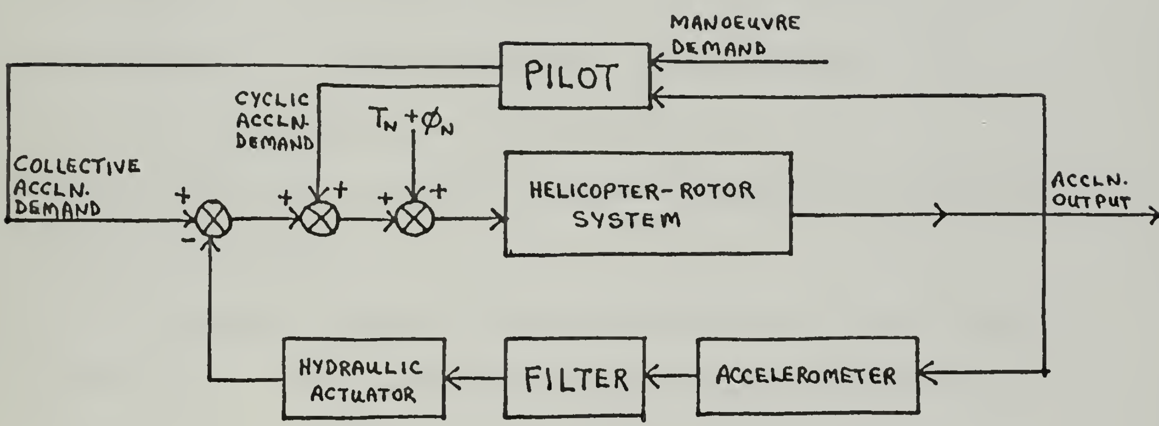


Fig. 7. Simplified Block Diagram of Servo Control System

The pilot's feedback path is shown above and is basically a low frequency path, quite separate from the actuator feedback which is basically a high frequency path. The significance of the pilot's path is explained later but for the present it can be left out of the

succeeding block diagrams which concern only the proposed vibration control system loop. The block diagram of Fig. 7 should be quite simple to realize, either on a model or on an actual helicopter. For the present, however, it must be dealt with theoretically. Therefore we must engage our thought processes in setting down the most realistic demands for each system component included above. Some sort of practical analysis may then follow, eventually leading to a physical realization. The choice of basic input to Fig. 7, i.e. collective acceleration demanded, is arbitrary. It could just as easily have been collective stick displacement; but for this analysis collective acceleration demanded is used.

Basically there are four transfer functions to be determined. To a sufficiently close approximation the accelerometer and the hydraulic actuator can each be described as second order systems and therefore having transfer functions of the type

$$\frac{\text{OUTPUT}}{\text{INPUT}} = \frac{\text{CONSTANT}}{1 + 2\zeta T s + T^2 s^2} \quad . \qquad (44)$$

Typical values of resonant frequencies and damping ratios will be suggested, to provide a basis for determining a closed loop transfer function for the total system. The resonant frequency of the accelerometer is taken to be 150 Hz, and that of the hydraulic actuator as 60 Hz. A damping ratio of 0.5 for each is assumed.

For the present, the 'filter' will be assumed to be a simple gain, K_F. This means we have not provided a way to block the D.C. level that the accelerometer senses, which is needed to allow the helicopter to have a good response to manoeuvre g. However, by using an accelerometer with a velocity pickoff, giving rise to a rate of change of acceleration output from the instrument, we shall block the D.C. acceleration level from being fed back, the effect of which is described by the inclusion of an S term in the numerator of expression (44). Now we may derive a transfer function for the helicopter rotor system.

It is a reasonable assumption that the blade collective pitch demanded, θ_D, is proportional to the vertical acceleration demanded, for steady flight conditions. Thus:

$$\theta_D = K_\theta \times \text{ACCELERATION}_{\text{DEMANDED}} \tag{45}$$

Similarly, the rotor output acceleration may be expected to be directly proportional to the blade coning angle, β_o.

In many very simplified analyses β_o is considered to be constant and directly proportional to lift, hence acceleration, which is also considered to be constant. However, we know that both β_o and acceleration are not just constant terms, by virtue of the description of flapping vibration in Chapter 2, due to periodic coning. Thus the vibration control loop of Fig. 7 can be redrawn in more detail in Fig. 8 as follows :

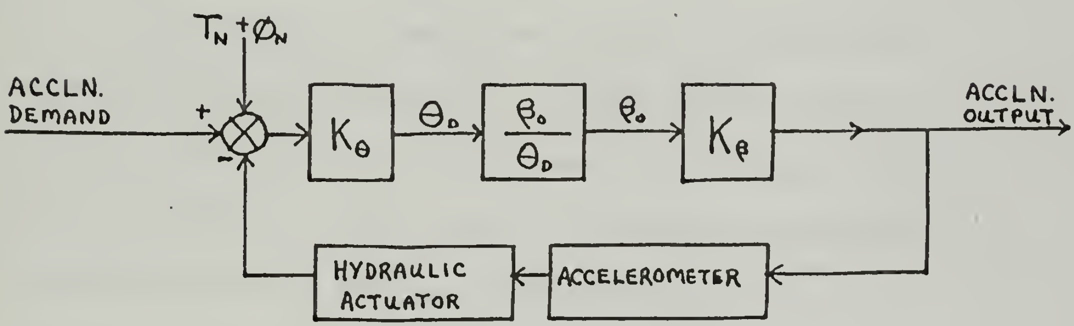


Fig. 8 Simplified Block Diagram of Servo Control System

From Fig. 8 we can see that a transfer function for β_o/θ_D is required. Fortunately one is available from the solution of the equations of motion in the longitudinal plane for a helicopter in steady forward flight [9]. In slightly different symbolic notation

than that of Ref. 9 we get

$$\frac{\beta_o}{-\theta_D} = \frac{\bar{d}_3 S^3 + \bar{d}_2 S^2 + \bar{d}_1 S + \bar{d}_o}{e_5 S^5 + e_4 S^4 + e_3 S^3 + e_2 S^2 + e_1 S + e_o}$$

$$+ \frac{(S_{11} M_{y\theta_D} - S_{21} Z_{\theta_D}) S + (T_{11} M_{y\theta_D} - T_{21} Z_{\theta_D})}{\bar{C}_2 S^2 + \bar{C}_1 S + \bar{C}_o} . \qquad (46)$$

Appendix A contains the definitions of all the coefficients of the terms in S in expression (46). It also contains sufficient numerical data, as given in Ref. 9, to enable sample values for the coefficients of the terms in S in expression (46) to be obtained.

The helicopter being considered in this case study weighs 5000 pounds, has a three-bladed rotor, a tip speed ratio, μ , of 0.30, and a rotor speed, Ω , of 20.3 rad/sec. Other data are listed in Appendix A. Performing all the necessary calculations we get the typical transfer function :

$$\frac{\beta_o}{-\theta_D} = \frac{(394 \times 10^{12}) S^3 + (45{,}175 \times 10^{12}) S^2 + (3680 \times 10^{12}) S + (256 \times 10^{12})}{(-1010 \times 10^{12}) S^5 + (-14 \times 10^{15}) S^4 + (-20.7 \times 10^{13}) S^3 + (-5108 \times 10^{12}) S^2 + (-625 \times 10^{12}) S + (-1500 \times 10^{12})}$$

$$+ \frac{(18.8 \times 10^6) S + (0.4 \times 10^6)}{(-2.235 \times 10^6) S^2 + (-31.45 \times 10^6) S + (-36.77 \times 10^6)} . \qquad (47)$$

This simplifies to

$$\frac{\beta_o}{\theta_D} = \frac{256[1 + 14.4S + 177S^2 + 1.54S^3]}{1500[1 + .416S + 3.4S^2 + 13.85S^3 + 9.35S^4 + 0.67S^5]}$$

$$+ \frac{0.4[1 + 47S]}{36.77[1 + 0.85S + .063S^2]} \quad . \tag{48}$$

Collecting the terms in (48), we get

$$\frac{\beta_o}{\theta_D} = \frac{0.181[1 + 4.3S + 180S^2 + 154S^3 + 51S^4 + 26.5S^5 + 1.9S^6]}{[1 + .416S + 3.4S^2 + 13.85S^3 + 9.35S^4 + 0.67S^5][1 + 0.85S + .063S^2]} \quad . \tag{49}$$

Now the numerator of (49) factorizes as follows :

$$\frac{\beta_o}{\theta_D} = \frac{0.181[1 + 3.5S + 177S^2 + 10S^3 + 32S^4][1 + .81S + .06S^2]}{[1 + .416S + 3.4S^2 + 13.85S^3 + 9.35S^4 + 0.67S^5][1 + 0.85S + .063S^2]} \quad . \tag{50}$$

Clearly, two of the quadratics are close enough to being equal that they may be cancelled. The quartic left in the numerator then factorizes into two quadratics while the real root in the denominator also factorizes out, leaving

$$\frac{\beta_o}{\theta_D} = \frac{0.181[1 + .0531S + .181S^2][1 + 3.447S + 176.6S^2]}{[1 + 1.557S][1 - 1.141S + 5.18S^2 + 5.73S^3 + 0.43S^4]} \quad . \tag{51}$$

The denominator quartic then factorizes as follows

$$\frac{\beta_0}{\Theta_D} = \frac{0.181[1+.0531S+.181S^2][1+3.447S+176.6S^2]}{[1+1.557S][1+.858S+.0629S^2][1-1.999S+6.83S^2]} \quad . \tag{52}$$

The higher frequency quadratics of expression (52) are not too important. Of the low frequency terms, the quadratic with negative damping is very conspicuous. Expression (52) represents a quite unstable system. In his analysis Nikolsky points out that β_0 is positively damped when considered by itself, but becomes unstable because of the effect of forward speed and attitude changes [9].

Nevertheless, pilots have been able to fly helicopters with this typical type of unstable transfer function for a long time. The reason they can do so is that they provide their own feedback loop around the forward speed and attitude control paths, keeping these variables within safe limits. Consequently they keep the coning angle β_0 from going unstable by providing the necessary control functions with the cyclic pitch control, rather than the collective pitch control.

Since the proposed vibration-control system provides feedback around the collective control only,

the closed loop transfer function obtained should still be of similar order and form as the original open loop transfer function. Then the closed loop transfer function should be compared with the open loop transfer function to determine if vibration reduction has been accomplished at high frequencies and to determine if the basic helicopter performance has been drastically modified.

Redrawing the block diagram, with the transfer function included, we get Fig. 9

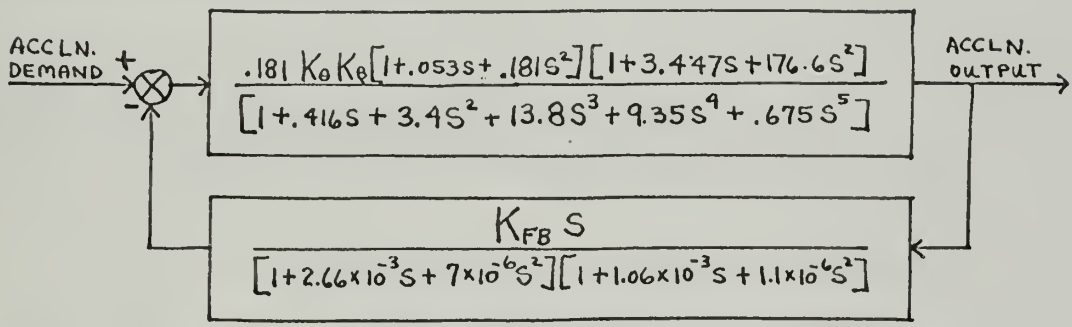


Fig. 9 Closed Loop Servo System With Typical Transfer Functions

The two quadratics in the feedback path of Fig. 9 are those representing the hydraulic actuator and accelerometer responses for the suggested damping and resonant frequencies. The feedback D.C. gain has been normalised to K_{FB}.

Now for the typical three-bladed rotor being considered, the predominant vibrations are in the range from about nine to 27 Hz (3/rev to 9/rev). The devices in the feedback path were specifically chosen to have high (above 50 Hz) resonant frequencies. Therefore the denominator of the feedback transfer function can be ignored up to 375 rad/sec when the hydraulic actuator begins to attenuate all feedback signals.

Now the steady state output/input of Fig. 9 is $0{\cdot}181\, K_\theta K_\rho$, and this is unity. Thus the closed loop transfer function becomes

$$\frac{\text{OUTPUT}}{\text{INPUT}} = \frac{\left[1+.053S+.181S^2\right]\left[1+3.45S+176.6S^2\right]}{\left[1+.416S+3.4S^2+13.8S^3+9.35S^4+.67S^5\right]+K_{FB}S\left[1+3.5S+177S^2+10S^3+32S^4\right]} \,. \tag{53}$$

The next consideration is to decide on how much feedback to use. As two possibilities, the closed loop transfer functions for $K_{FB} = 0{\cdot}10$ and $1{\cdot}0$ will be computed. These are, respectively

$$\left.\frac{\text{OUTPUT}}{\text{INPUT}}\right|_{K_{FB}=0.10} = \frac{\left[1+.053S+.181S^2\right]\left[1+3.45S+176.6S^2\right]}{\left[1+.516S+3.75S^2+31.5S^3+10.35S^4+3.87S^5\right]} \,, \tag{54}$$

$$\left.\frac{\text{OUTPUT}}{\text{INPUT}}\right|_{K_{FB}=1.0} = \frac{[1+.053S+.181S^2][1+3.45S+176.6S^2]}{[1+1.416S+6.9S^2+191S^3+19.35S^4+32.7S^5]} \quad . \tag{55}$$

Now the real roots factorize from the denominators of expressions (54) and (55) as follows

$$\left.\frac{\text{OUTPUT}}{\text{INPUT}}\right|_{K_{FB}=0.10} = \frac{[1+.053S+.181S^2][1+3.45S+176.6S^2]}{[1+2.795S][1-2.279S+10.12S^2+3.21S^3+1.39S^4]} \quad , \tag{56}$$

$$\left.\frac{\text{OUTPUT}}{\text{INPUT}}\right|_{K_{FB}=1.0} = \frac{[1+.053S+.181S^2][1+3.45S+176.6S^2]}{[1+5.81S][1-4.394S+32.45S^2+2.36S^3+5.64S^4]} \quad . \tag{57}$$

The quartics in expressions (56) and (57) can then be factorized into two quadratics, so that we can compare the following expressions with that of expression (52).

$$\left.\frac{\text{OUTPUT}}{\text{INPUT}}\right|_{K_{FB}=0.10} = \frac{[1+.053S+.181S^2][1+3.45S+176.6S^2]}{[1+2.795S][1+.3263S+.128S^2][1-2.61S+10.84S^2]} \tag{58}$$

$$\left.\frac{\text{OUTPUT}}{\text{INPUT}}\right|_{K_{FB}=1.0} = \frac{[1+.053S+.181S^2][1+3.45S+176.6S^2]}{[1+5.81S][1+.0957S+.1725S^2][1-4.49S+32.7S^2]} \quad . \tag{59}$$

Referring to expression (52), except for the constant term, it represents the forward path, or open-loop transfer function of a typical helicopter. True it is unstable, but as stated earlier pilots learn to fly it without undue difficulty. Looking at two of the quadratics, $\dfrac{[1+.053S+.181S^2]}{[1+.858S+.0629S^2]}$, one can see that there is no attenuation of higher frequency inputs, but rather that there is even some amplification. The first order lag term is of the order of 1·5 seconds. The divergent oscillation in expression (52) contains the exponential coefficient, $\xi\omega_N$, equal in magnitude to 0·146.

Looking at the closed-loop transfer functions of expressions (58) and (59) it can be seen that the higher frequency quadratics tend to become equal, as more feedback is used, i.e. $\dfrac{[1+.053S+.181S^2]}{[1+.0957S+.1725S^2]}$ very nearly cancels, and it will tend to completely cancel if the feedback is increased a little more. Thus, by using rate of change of acceleration feedback,

this proposal reduces the tendency of this particular helicopter to transmit high frequency inputs, hence some vibration reduction has been shown to result.

The divergent oscillations are still indicated as being present in expressions (58) and (59), where the exponential coefficients are equal to 0.12 and .0686 respectively. Again, by employing this proposed form of feedback, a beneficial effect is produced, namely a decrease in the rate at which the divergency occurs.

Finally, comparing the remaining term in the denominator, i.e. the first-order lag, one sees that the time lag has increased to 2.8 and 5.81 seconds, for expressions (58) and (59) respectively. This appears to represent a more sluggish control response and is not a beneficial effect of using this type of feedback.

The proposal has been presented in its simplest form. Because of the inherently unstable behaviour of the closed-loop transfer functions only the time responses to step function inputs can be plotted. However, it was felt that the time taken to do this would be excessive in view of the limited information

it would yield.

Summarizing the theoretical results, vibration reduction is successfully accomplished, and at the same time the effect of the divergent oscillation is reduced. The size of the first-order lag is, however, increased. The proposal, as presented, appears feasible, and it has probably come to a stage where it could now be put on a simulator, say an analogue computer, for more comprehensive study. However, the intention of this thesis was to determine the feasibility of applying this proposed vibration reduction technique to helicopters in general, and a simulation would only be useful for a particular helicopter.

The preceding work has indicated that the effect of the proposed instrumentation is merely to modify the rate of response, but not to alter, structurally, the form of the helicopter dynamics.

5. DISCUSSION

Now that the system has been described further discussion is necessary. One might wonder if the system would work on every helicopter. This would depend on the type of helicopter and on the type of forward path transfer function typifying a chosen model, both of which are quite variable. Before one instals the proposed system to flight test it there are further problems to consider.

Undoubtedly the most important issue on which the engineer has to satisfy himself is that the system is safe for flight. This he can determine only after his model, or computer simulation, has been proved to be correct and safe for all probable flight conditions.

As an example, let us consider whether this proposed system would apply to a rigid rotor. The rigid rotor has no flapping hinge. However, the blades of a rigid rotor still flap, but due to blade flexibility or bending. Nevertheless, the system should still work on rigid rotor helicopters, because they do have a collective pitch control and they do vibrate. The analysis would change somewhat in that

the rotor transfer function would be different.

Also, a more comprehensive study would require an analysis that sought to determine how much cross-coupling existed among resonant flapping or bending modes or between both. And then there is always the possibility that even if the cyclic harmonic vibrations are attenuated, vibrations at new resonant frequencies may appear, for instance, torsional vibration of the rotor. As Shapiro points out,

> The other practical significance of the torsional vibration of the rotor as a whole is the free oscillation excited by single torsional impulses, such as sudden increments of collective feathering. Such impulses always excite mainly the fundamental (lowest frequency) mode of free vibrations and can in practice lead to large amplitudes [11].

Is it equally valid to assume that the system will work for all flight conditions that a helicopter can encounter, once it is proven feasible for a helicopter model? Again the answer would require much more detailed analysis than presented in this paper. However this opens the door to another possibility.

As was previously mentioned, a filter may be

necessary in the feedback path. It would be nice to improve the control response and attenuate vibrations even more. This might be done by using filter techniques. Remember for the simple proposal, the feedback used was basically $K_{FB}S$, representing about the crudest filter available. The case considered was for high speed forward flight, and as speed increases so does the vibration level. Therefore it would be nice to make the feedback a function of forward speed.

To make this system work for every flight condition may require an adaptive filter. The adaptive filter could be a function of any one or more of the following: 1) rotor RPM 2) forward or lateral horizontal velocity 3) vertical velocity 4) air density 5) gross weight. Another possibility is to use a tuneable filter, controlled by the pilot, allowing him to vary the feedback gain and phase for any given flight conditions.

Better low frequency attenuation may be called for in the feedback path, than that available from the $K_{FB}S$ term. This might mean an even more complex filter would have to be included. At any result the proposal has reached a point where it can be simulated and filter

techniques applied in the feedback path. The end result of the feedback filter should be two-fold, one, to reduce higher frequency vibrations even more, and two, to restore the first-order lag to a value close to the open-loop case. It should not be the object of the filter to stabilize the divergent oscillation, but if some beneficial effects on the divergence come about they would be welcome.

Before other considerations are brought out, safety must be mentioned. If the system should fail, the pilot should have about the same response from his collective control as he does without the system installed. Some degradation of control is acceptable, and usually much cheaper. The reliability of the collective pitch control with the proposed system installed would have to be no higher than existing collective control reliability.

The number of blades on the rotor has two distinct effects also. The more blades there are the greater the power and size of the hydraulic actuator. In a helicopter weight is very critical; therefore if the actuator and associated hydraulics get too big and too

heavy the weight penalty may override the vibration reduction gained. Secondly, the more blades there are, the higher are the vibration frequencies, i.e. two-bladed rotor (2/rev), three-bladed rotor (3/rev). This means that as the number of blades increases, the bandpass of the feedback path, due to the hydraulic servo performance, limits the upper frequencies, and harmonics, at which vibration signals are fed back.

Another problem that will have to be more fully investigated is the effect this system will have on the in-plane vibrations. As can be determined from expressions (25) and (26), and Fig. 5, there is definitely a cross-coupling of forces. The reduction in the vertical acceleration level should reduce in-plane vibrations, but since these also cross-couple with torsional vibration of the rotor the complete system would require investigation. Because of all these possible interactions the system may require a separate actuator to each blade, so as to provide phase control over the feedback. If so, this would be a cyclic modification and the effects on cyclic control would have to be studied.

Experiments are also going on with other types

of rotors. For example, the Cheeseman rotor, which gets its lift by varying the circulation on it, might be adaptable to this proposal. Of course, the hydraulic jack couldn't be used. But the accelerometer output could go to a control valve, and this control valve could, in turn, regulate the amount of engine bleed air blowing out the back of the Cheeseman rotor, and hence the circulation and lift. The problems to consider here lie in pneumatics mainly, such as air compressibility and the resonant frequency of the control valve.

Finally one might ask why this vibration reduction system was proposed in the first place. The initial reason that led to this system proposal was to reduce helicopter and pilot fatigue. A second reason that came out as the proposal developed was to improve human comfort. Helicopters can be made to withstand severe vibrations, but people have a vibration tolerance that is much lower.

Brandt feels that there is probably a universal agreement that the human comfort criteria used today are inaccurate and quite often inadequate [2]. As an example he refers to the U.S. Military Specification,

General Requirements for Helicopter Flying and Ground Handling Qualities, (MIL - H - 8501A, Sept 1961) [14]. Ref. 14 states that 'in general, throughout the design flight envelope, the helicopter shall be free of objectionable shake, vibration, or roughness'. It then gives specific vibration levels and frequencies for various flight conditions, assumed to be applicable to all helicopters or rotary-winged aircraft.

Since 1961 there have been drastic changes in the size, speed, weight, and vibration characteristics of rotary-winged aircraft. Even with human comfort criteria fully optimised, there seems to be a need for redefining acceptable vibration levels to fit individual helicopters or V/STOL aircraft classes. If Brandt is correct, possibly a human factors research team should explore Ref. 14 and others like it, conduct a new criteria research, and make new proposals.

The paper by Cdr. J.R. Williford on the special problems of anti-submarine warfare helicopters strengthens this suggestion [16]. In his paper he points out problem difficulties such as the 'inability to eliminate - sometimes even reduce - vibration in all regimes of flight without resorting to a vibration

"sink" or absorber'. If specifications such as Ref. 14 were changed it may force designers to incorporate new vibration controlling systems into new helicopters at the design stage rather than as expedients after the basic helicopter is produced.

What would a system like the one proposed cost to develop? This would take a detailed time-cost study, but Brandt stated estimates of several hundred thousand dollars to develop new ideas as mentioned in Ref. 2. This proposed system may take more or less, but if in the end not only are vibration levels brought way down but also increased rotor life is gained, the whole proposal would be beneficial.

Does the system provide any gust alleviation? Again, this could be investigated on a simulator. Finally, does the system unload the rotor and increase its useful life? This again would have to be investigated, probably by using strain gauge techniques on the rotor before and after the system is installed.

6. SUGGESTIONS FOR FURTHER WORK

For a further and more detailed evaluation of the proposed vibration control system the author considers the following work should prove fruitful. The first suggestion would be to obtain the transfer function for β_o/θ_D for three different helicopters, varying greatly in size, weight, and rotor configuration. For these three helicopters then obtain the transfer functions, β_o/θ_D , for hovering, normal cruise, and fast cruise flight conditions. One could then analyse the forward path transfer functions and determine if they are of reasonably the same type.

Next one could employ various amounts of feedback as previously proposed to see if vibrations would be reduced for the various helicopters and/or flight conditions. This is merely the logical extension of the previous specific case to a broader range of designs.

The next approach is to determine a more sophisticated filter to put in the feedback path so as to reduce vibrations more effectively without increasing the first-order lag. A simulator could now be used during these studies. Initial analogue

computer studies could be kept simple and linear, and responses to various disturbances could be plotted in the time domain.

If these responses looked promising the problem could be made more difficult. Some of the assumptions mentioned in Chapter 2 could be rejected and some more realistic simulation could be employed. For example, the induced velocity would not have to be assumed constant and uniform. It could be variable and even of a non-linear form. The fact that there is cross-coupling between the vertical and in-plane forces could be included too.

Another approach that is worthy of thought would be to use predictive sampling techniques. The rotor rotates at nearly a fixed speed, and the vibrations occur at nearly fixed frequencies and amplitudes, for any given steady flight condition. Therefore, vibration levels and frequencies could be recorded for, say, five rotor rotations, averaged, and then fed back to the collective pitch control in opposition at precisely the correct rotor blade azimuthal position at which they originally occurred. One might call this a measure-predict-correct system.

Once the system has been proved safe theoretically, it remains to prove it on an actual helicopter. The development of this system might progress faster and just as safely if one went right to a helicopter and installed it. Tethering the helicopter in known wind conditions would allow the actual system to be tested with the rotor running, and a real pilot could move the collective pitch control. Not only would the pilot's comments be available but also complete rotor blade motion studies could be made and compared with these same studies made before the system was actually installed. The gain in the feedback path could be left variable in order to study a broader band of control conditions.

Another possibility is to make a small real model of a helicopter with rotor blades and collective pitch control and instal the proposed vibration control system. This could then be put in a wind tunnel and tested under various flight conditions. It would provide yet another way to compare theory with physical results.

The increase in the first-order time lag and the reduction in the divergency term means a modification

to the controllability of the helicopter. Therefore it might prove to be very fruitful to introduce a pilot loop in the simulator studies to determine when an unstable system is more or less controllable.

Another consideration that simulator studies could investigate would be the vibrations induced by causes other than the rotor. These would include engine vibrations, tail-rotor vibrations, and vibrations resulting from the installation of small stability augmentors, such as horizontal stabilizers.

The proposed system should be investigated to see if it might also prove feasible for multi-rotor helicopters. The fundamental concept should remain the same, with different requirements for instrumentation and feedback. Whether one accelerometer would suffice, or whether an accelerometer would be required for each rotor, would have to be determined.

The horsepower required for the hydraulic actuator must come from the engine. This means there would be a reduction in the maximum lift-off weight of the helicopter. Thus the weight added to the helicopter by the proposed servo control system would have to be very carefully measured in order to determine if the

helicopter could still perform all required missions with the added weight under reduced power conditions. The above mentioned suggestions are by no means all of the available suggestions for future work on this proposal. They are considered to be among the most important however.

7. CONCLUSIONS AND ACKNOWLEDGEMENTS

The system as proposed for this one simple case appears feasible. Further theoretical work is necessary, followed by some type of simulation, either on a computer or on a scale model. More detailed design of the present feedback path appears to be desirable to obtain a more complex filter which will assist in successfully eliminating the vertical vibration, but which will leave the first-order lag term more nearly as it was without vibration control feedback.

The author takes this opportunity to thank Professor J.A.J. Bennett of the College of Aeronautics for his personal attention in the form of notes, discussion, and constructive criticism during the writing of this proposal. Finally, this author owes a debt of gratitude to Mr. K.C. Garner of the College of Aeronautics for supervising this thesis from start to finish. Mr. Garner originated this proposal, and he provided this most interesting thesis topic in the first place. Without Mr. Garner's patience and guidance this thesis would not have been done.

8. BIBLIOGRAPHY

1. Bennett, J.A.J. Rotary-Wing Aircraft. Aircraft Engineering, May, 1940.

2. Brandt, D.E. Vibration Control In Rotary-Winged Aircraft. AGARD Conference Proceedings No. 7, Helicopter Developments, Jan. 10-14, 1966.

3. Carroll, R.P., and others, An Integrated Hydraulic Control System Combining Boost, Control Feel, Stability Augmentation, And Gust Alleviation, 19th Annual National Proceedings of the American Helicopter Society, 1963.

4. Drees, Jan M., and Lynn, Robert R. The Promise of Compounding. Same source as Ref. 2.

5. Jones, J.P. The Vibration Characteristics of a Circulation-Controlled Rotor Considering Rotor Blade Elasticity. ARC 26454 United Kingdom Sept. 1963.

6. Jones, J.P. The Use of an Analogue Computer To Calculate Rotor Blade Motion. Journal of the American Helicopter Society, vol. 9, April, 1964.

7. Kawalkiewicz, Frank S., and others, Helicopter Vibration Indicator. U.S. Army TRECOM Tech. Report 63-44 October 1963.

8. Miller, R.H. Rotor Blade Harmonic Air Loading IAS Paper No. 62-82 New York Jan. 1962.

9. Nikolsky, A.A. Helicopter Analysis. John Wiley and Sons Inc., New York 1951.

10. Seris, H., and Auffret, R. Measurement of Low-Frequency Vibrations in Big Helicopters and Their Transmission to the Pilot. AGARD Conference Proceedings No. 2, Aerospace Medical Panel, 2-6 Sept. 1965.

11. Shapiro, J. Principles of Helicopter Engineering. Temple Press, London, 1955.

12. Sherby, S.S. Design Philosophy of the OH-5A Helicopter. Proceedings of the 19th Annual Forum of the American Helicopter Society, 1963.

13. Smollen, L.E., Marshall, P., and Gabel, R. A Servo Controlled Rotor Vibration Isolation System for the Reduction of Helicopter Vibration. IAS Paper No. 62-34 New York, Jan. 1962.

14. U.S. Military Specification, Helicopter Flying and Ground Handling Qualities; General Requirements for. MIL-H-8501A, 7 Sept. 1961.

15. Wernicke, Rodney K., and Drees, Jan M. Second Harmonic Control. 19th Annual National Proceedings of the American Helicopter Society, 1963.

16. Williford, J.R. Cdr. USN Special Problems In Helicopter Handling Qualities As Influenced By Anti-Submarine Warfare Requirements. Same source as Ref. 2.

APPENDIX A

Typical Formulae and Helicopter Data

$\bar{d}_3 = \bar{a}_{2\mu} S_{15}$	394×10^{12}
$\bar{d}_2 = \bar{a}_{1\mu} S_{15} + \bar{a}_{1\alpha} T_{15} + \bar{a}_{2\mu} S_{16}$	$45{,}175 \times 10^{12}$
$\bar{d}_1 = \bar{a}_{0\mu} S_{15} + \bar{a}_{0\alpha} T_{15} + \bar{a}_{1\mu} S_{16} + \bar{a}_{1\alpha} T_{16}$	$3{,}680 \times 10^{12}$
$\bar{d}_0 = \bar{a}_{0\mu} S_{16} + \bar{a}_{0\alpha} T_{16}$	256×10^{12}
$S_{15} = Z_{\mu x} S_{21} - M_{y\mu_x} S_{11}$	8.30×10^{6}
$T_{15} = Z_{\alpha_1} S_{21} - M_{y\alpha_1} S_{11}$	18.80×10^{6}
$S_{16} = Z_{\mu_x} T_{21} - M_{y\mu_x} T_{11}$	2.13×10^{6}
$T_{16} = Z_{\alpha_1} T_{21} - M_{y\alpha_1} T_{11}$	$.40 \times 10^{6}$
$e_5 = \bar{b}_3 \bar{c}_2$	-1010×10^{12}
$e_4 = \bar{b}_3 \bar{c}_1 + \bar{b}_2 \bar{c}_2$	$-14{,}050 \times 10^{12}$
$e_3 = \bar{b}_3 \bar{c}_0 + \bar{b}_2 \bar{c}_1 + \bar{b}_1 \bar{c}_2$	$-20{,}730 \times 10^{12}$
$e_2 = \bar{b}_2 \bar{c}_0 + \bar{b}_1 \bar{c}_1 + \bar{b}_0 \bar{c}_2$	$-5{,}108 \times 10^{12}$
$e_1 = \bar{b}_1 \bar{c}_0 + \bar{b}_0 \bar{c}_1$	-625×10^{12}
$e_0 = \bar{b}_0 \bar{c}_0$	$-1{,}500 \times 10^{12}$

$\bar{c}_2 = S_{11} S_{22} - S_{12} S_{21}$	$-[illegible] \times 10^6$
$\bar{c}_1 = S_{11} T_{22} + T_{11} S_{22} - S_{12} T_{21}$	-31.45×10^6
$\bar{c}_0 = T_{11} T_{22}$	-36.77×10^6
$S_{11} = Z_{\dot{\mu}_z}$	75×10^3
$S_{12} = Z_{\dot{\rho}_0}$	$-2,[illegible]$
$S_{21} = M_{y\dot{\mu}_z}$	34.3
$S_{22} = M_{y\dot{\rho}_0}$	-32.1
$T_{11} = Z_{\mu_z}$	89.23×10^3
$T_{21} = M_{y\mu_z}$	873
$T_{22} = M_{y\rho_0}$	-412
$\bar{a}_{2\mu} = \bar{H}_{\theta_0} N_{22} - \bar{M}_{y.\theta_0} N_{12}$	47.52×10^6
$\bar{a}_{1\mu} = \bar{H}_{\theta_0} P_{22} - \bar{M}_{y.\theta_0} P_{12}$	-2.97×10^6
$\bar{a}_{0\mu} = \bar{H}_{\theta_0} Q_{22} - \bar{M}_{y.\theta_0} Q_{12}$	90.60×10^6
$\bar{a}_{1\alpha} = P_{11} \bar{M}_{y.\theta_0}$	2410×10^6
$\bar{a}_{0\alpha} = Q_{11} \bar{M}_{y.\theta_0} - Q_{21} \bar{H}_{\theta_0}$	158×10^6
$\bar{b}_3 = P_{11} N_{22}$	435×10^6

$\bar{D}_2 = [illegible]_{11} P_{22} + [illegible]_{11} [illegible]_{22} - N_{12} [illegible]_{21}$ 151.9 x 10^6

$\bar{D}_1 = [illegible]_{11} Q_{22} + [illegible]_{11} P_{22} - [illegible]_{12} [illegible]_{21}$ -13 x 10^6

$\bar{D}_0 = [illegible]_{11} [illegible]_{22} - [illegible]_{12} [illegible]_{21}$ 40.3 x 10^6

$P_{11} = \bar{I}\dot{\mu}_x$ 75.5 x 10^3

$P_{12} = \bar{H}\dot{\alpha}_1$ 21?

$P_{22} = \bar{M}_{y_o\dot{\alpha}_1}$ 1330

$Q_{11} = \bar{H}\mu_x$ 6290

$Q_{12} = \bar{H}\alpha_1$ -2860

$Q_{21} = \bar{M}_{y_o\mu_x}$ 14,924

$Q_{22} = \bar{M}_{y_o\alpha_1}$ -307

$N_{12} = \bar{H}\ddot{\alpha}_1$ -970

$N_{22} = \bar{M}_{y_o\ddot{\alpha}_1}$ 5775

$\bar{M}_{y_o\theta_o} = M_{y_o\theta_o} - M''_{y_o\theta_o} M$ 31,943

$\bar{M}_{y_o\alpha_1} = M_{y_o\alpha_1} - M''_{y\alpha_1} M$ -307

$\bar{M}_{y_o\dot{\alpha}_1} = M''_{y\dot{\alpha}_1} M$ 1330

$\bar{M}_{y_o\ddot{\alpha}_1} = M_{y_o\ddot{\alpha}_1}$ 5775

$\bar{M}_{y_0\mu_x} = M_{y_0\mu_x} - M''_{y\mu_x} M$	14,994
$M = M_{y_0 a_1} / M_{y a_1}$	-32.69
$\bar{H}_{\theta_0} = H_{\theta_0} - M''_{y\theta_0} H$	2,370
$\bar{H}_{\alpha_1} = H_{\alpha_1} - M''_{y\alpha_1} H$	-2,365
$\bar{H}_{\dot{\alpha}_1} = M''_{y\dot{a}_1} H$	212
$\bar{H}_{\ddot{\alpha}_1} = H_{\ddot{\alpha}_1}$	-970
$\bar{H}_{\mu_x} = H_{\mu_x} - M''_{y\mu_x} H$	6,290
$\bar{H}_{\dot{\mu}_x} = H_{\dot{\mu}_x}$	75,500
$H = H_{a_1} / M''_{y a_1}$	-5.23
$M''_{y\theta_0} = \frac{\gamma_1 \Omega^2}{8}\left(1 + \frac{3}{2}\mu^2\right)$	740
$M''_{y a_1} = \frac{-\gamma_1 \Omega^2}{8}\left(1 - \frac{\mu^2}{2}\right)$	-626
$M''_{y\dot{a}_1} = -2\Omega$	-40.6
$M''_{y\alpha_1} = -\frac{\gamma_1 \Omega^2}{8}\left(1 + \frac{3}{2}\mu^2\right)$	-740
$M''_{y\mu_x} = \frac{\gamma_1 \Omega^2}{8}\left[\frac{8}{3}\theta_0 + 2\lambda_a - 3\mu(\alpha_1)_0 + \mu(a_1)_0 + 2(\mu_z)_0 - 3\mu B_1\right]$	186
$M_{y\theta_0} = \frac{\gamma_1 \Omega^2}{6}\mu$	262
$M_{y\alpha_1} = -\frac{\gamma_1 \Omega^2}{6}\mu$	-262

$$M_{y\mu_x} = \frac{\gamma_1 \Omega^2}{8}\left[2\theta_0\mu - \frac{4}{3}(\alpha_1)_0 - \frac{4}{3}B_1\right] \qquad -115$$

$$M_{y\beta_0} = -\Omega^2 \qquad -412$$

$$M_{y\dot{\beta}_0} = -\frac{\gamma_1 \Omega}{8} \qquad -32.1$$

$$M_{y\mu_z} = \frac{\gamma_1 \Omega^2}{6} \qquad 273$$

$$M_{y\dot{\mu}_z} = M_S \Omega R / I_1 \qquad 34.3$$

$$M_{y_0\theta_0} = hH_{\theta_0} + Z_{\theta_0}\left[\bar{h} + h(\alpha_1)_0\right] \qquad 15,200$$

$$M_{y_0a_1} = hH_{a_1} \qquad 20,462$$

$$M_{y_0\alpha_1} = hH_{\alpha_1} - (C_{mf} - C_t)\mu^2 - \frac{a_3}{2}\bar{h}\mu + \frac{a_3}{2}h\left[2\theta_0\left(\frac{1}{3} + \frac{\mu^2}{2}\right) + \lambda_a + (\mu_z)_0 - \mu B_1 - 2\mu(\alpha_1)_0\right] \qquad 7,692$$

$$M_{y_0\ddot{\alpha}_1} = I_{y_0} \qquad 5,775$$

$$M_{y_0\mu_x} = hH_{\mu_x} - 2hD_{S_0}\mu\left(\frac{\Omega R}{100}\right)^2\frac{\rho}{\rho_0} + \frac{a_3}{2}\left[2\theta_0\mu - B_1 - (\alpha_1)_0\right]\left[\bar{h} + h(\alpha_1)_0\right] + (C_{mf} - C_t)\left[(\mu_z)_0 + \lambda_a - 2(\alpha_1)_0\mu\right] - 2C_t\theta_t\mu \qquad 7,844$$

$$H_{\theta_0} = -\frac{a_3}{4}\left[\lambda_a + (\mu_z)_0 - (a_1)_0\mu\right] \qquad -1024$$

$$H_{\alpha_1} = -H_{\theta_0} \qquad 1024$$

$$H_{a_1} = \frac{a_3}{4}\left\{\left(\frac{4}{3}\theta_0 + 3\lambda_a\right) - \mu\left[B_1 + (\alpha_1)_0 - 2(a_1)_0\right] + 3(\mu_z)_0\right\} \qquad 3274$$

$$H_{\ddot{\alpha}_1} \quad -\bar{m}h \qquad -970$$

$$X_{\mu_x} = \frac{a_3}{4}\left\{2\left(\frac{\delta}{a} - \theta_0 \lambda_a\right) - (a_1)_0\left[B_1 + (\alpha_1)_0 - (a_1)_0\right] + (\beta_0)_0^2 - 2(\mu_z)_0\theta_D\right\} + 2D_{S_0}\mu\left(\frac{\Omega R}{100}\right)^2\frac{\rho}{\rho_0} \qquad 5320$$

$$X_{\dot{\mu}_x} = \bar{m}\Omega R \qquad 75{,}500$$

$$Z_{\theta_D} = \frac{a_3}{2}\mu \qquad 26{,}300$$

$$Z_{\alpha_1} = -\frac{a_3}{2}\mu \qquad -26{,}300$$

$$Z_{\mu_x} = \left[\theta_0\mu - \frac{B_1}{2} - \frac{(\alpha_1)_0}{2}\right]a_3 + \left[(\mu_z)_0 + \lambda_a\right]D_{S_0}\left(\frac{\Omega R}{100}\right)^2\frac{\rho}{\rho_0} \qquad -9{,}350$$

$$Z_{\dot{\beta}_0} = \left[m_2 R/3\right](-1) \qquad -2{,}880$$

$$Z_{\mu_z} = \frac{a_3}{2} + \mu\left(\frac{\Omega R}{100}\right)^2\frac{\rho}{\rho_0}D_{S_0} \qquad 89{,}200$$

$$Z_{\dot{\mu}_z} = \bar{m}\Omega R \qquad 75{,}500$$

$$a_3 = a a_2 \Omega R \qquad 175{,}560$$

$$a_2 = \tfrac{1}{2}\rho b c \Omega R^2 \qquad 62.7$$

$$(\mu_x)_0 \cong \mu \qquad 0.3$$

$\bar{h}$ = Longitudinal distance of the c.g. of the aircraft from the shaft (positive when aft of axis of rotation) 0

$$\gamma_1 = c\rho a R^4/I_1 \qquad 12.65$$

$\bar{m}$ = Mass of aircraft in slugs 155

D_{s_0} = Structural drag at 100 ft/sec lbs	200
ΩR = ft/sec	487
$\rho = \rho_0$ slugs per ft^3	.002378
a = Blade lift curve slope	5.73
Ω = Rotor rotational speed rad/sec	20.3
R = Rotor blade radius ft	24
θ_0	.172
β_0	.1431
$(a_1)_0$	-0.1109
$(\mu_z)_0$	-0.001123
$(\alpha_1)_0$	0.131
B_1	0.078213
λ_a	-.088
h = Distance from c.g. to hub center	6.25
$(C_{m\beta} - C_t) = 0$ $\quad C_t \doteqdot$	96,500
$M_s = \int_0^R mrdr$	38.1
I_1 = Mom. of inertia of a blade about Y axis slug-ft^2	540

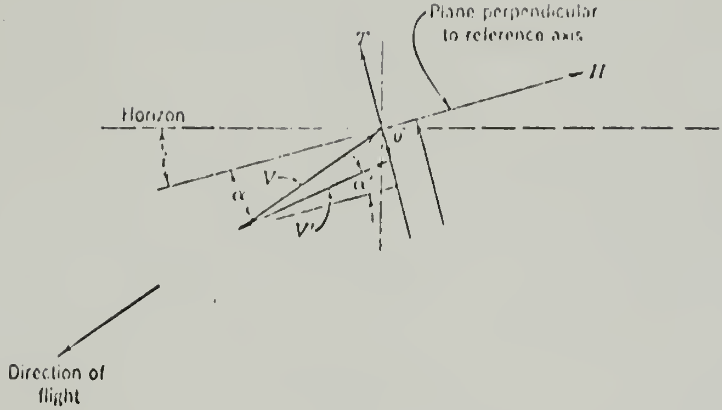

Plane perpendicular
to reference axis
T
H
Horizon
V
α
V'
Direction of
flight

Fig. 1

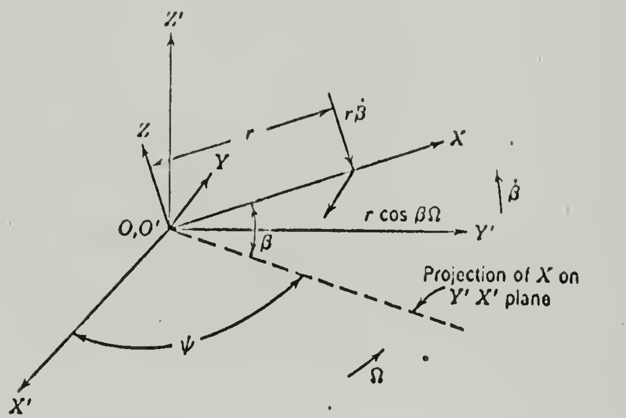
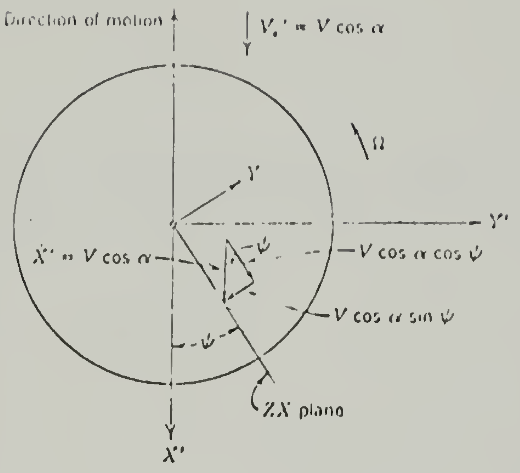
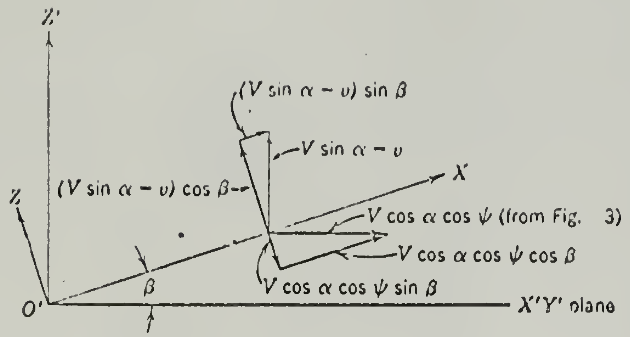

Z'
Z
Y
r
rβ̇
X
β̇
0,0'
r cos βΩ
Y'
β
Projection of X on
Y' X' plane
ψ
Ω
X'

Fig. 2



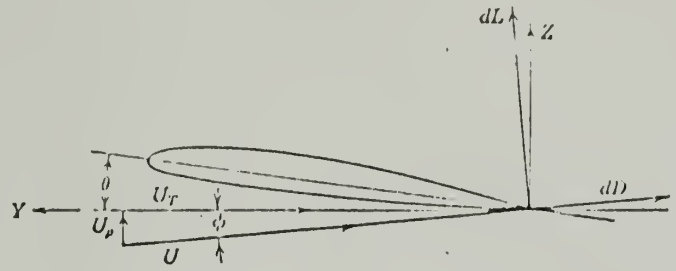
Direction of motion
$V_z' = V \cos \alpha$
Ω
Y
Y'
$X' = V \cos \alpha$
$V \cos \alpha \cos \psi$
$V \cos \alpha \sin \psi$
ψ
ZX plane
X'

Fig. 3



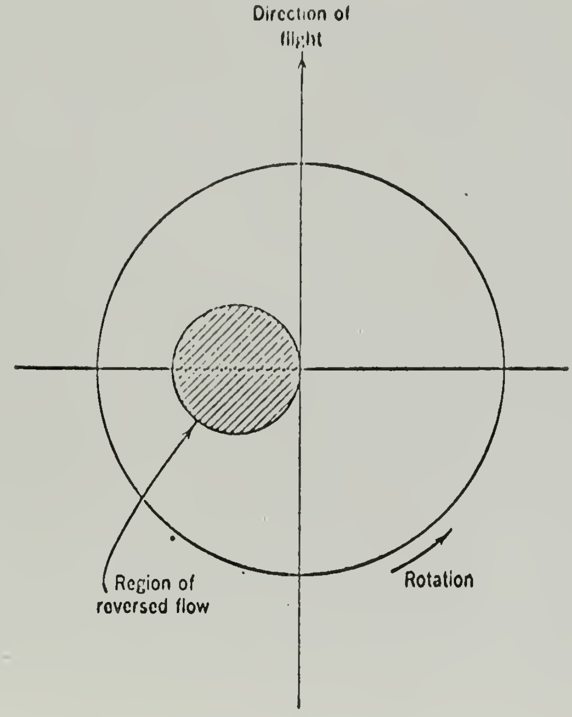
Z'
$(V \sin \alpha - v) \sin \beta$
$V \sin \alpha - v$
X
$(V \sin \alpha - v) \cos \beta$
Z
$V \cos \alpha \cos \psi$ (from Fig. 3)
$V \cos \alpha \cos \psi \cos \beta$
$V \cos \alpha \cos \psi \sin \beta$
β
O'
$X'Y'$ plane

Fig. 4



dL
Z
dD
Y
θ
U_T
φ
U_P
U

Fig. 5



Direction of flight
Region of reversed flow
Rotation

Fig. 6